

# A Framework to Guide a Research-based Approach to Teacher Education Materials



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## **Preparing to teach mathematics with technology: A framework to guide a research-based approach to teacher education materials**

Within the United States, the National Council of Teachers of Mathematics (NCTM, 2000) has long advocated that “technology is essential in teaching and learning mathematics; it influences what is taught and enhances students’ learning” (p. 24). Most recently, the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) adopted by most states in the U.S. claim, “mathematically proficient students ... are able to use technological tools to explore and deepen their understanding of concepts” (p. 7). And while many organizations and individuals purport and support the benefits of the use of technology to learn mathematics, many teachers are unsure about how to use technology in ways that promote students’ conceptual learning (e.g., Ertmer, 2005). Teacher preparation and professional development can assist teachers in developing their abilities to use technology in this way. Both the NCTM and the Association of Mathematics Teacher Educators have released position statements (AMTE, 2006, 2009; NCTM, 2011) emphasizing the importance of mathematics teacher preparation programs in providing appropriate experiences to create a workforce of teachers who can engage 21st century learners in using powerful technologies for learning and doing mathematics.

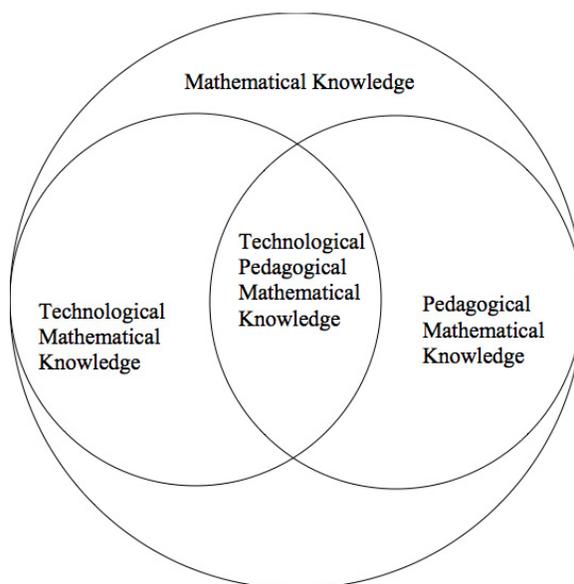
While it is evident that teacher preparation programs need to focus on improving teachers’ abilities to effectively integrate technology in mathematics classrooms, there are great differences in the ways teacher education programs do so. Teachers may learn to use technology in an education course, a mathematics course, or an educational technology course (Kersaint, Horton, Stohl, & Garofalo, 2003; Leatham, 2006). Each type of course provides different blends of attention to pedagogy, content, and technology. Evidence suggests a model that integrates pedagogy, content, and technology is more effective for preparing teachers to use technology in classrooms (Lee & Hollebrands, 2008; Niess, 2005; Suharwoto, 2006) and this is the approach we have taken in the development of the PTMT materials.

### **Framework and Guiding Principles**

Whether the use of technology will support or impede students’ learning depends on teachers’ decisions when using technology tools. These decisions are generally informed by teachers’ knowledge of (a) mathematics, (b) technology, and (c) pedagogy. Consider the following example that illustrates these three components of teacher knowledge in a lesson on exploratory data analysis. A teacher who is teaching a lesson focusing on comparing two different distributions needs to know how to compute and reason about measures of center and spread (knowledge of content), use technology to create representations of a distribution (knowledge of technology specific to the content), and design activities that align with approaches students may take when asked to compare two different distributions (knowledge of pedagogy specific to the content). The intersection of these three forms of knowledge has been identified as technological pedagogical content knowledge (TPCK) (Koehler & Mishra, 2005; Mishra & Koehler, 2008; Niess, 2005), a type of knowledge several authors have characterized as necessary for teachers to understand how to use technology effectively to teach specific subject matter.

Shulman’s (1986) construct of teachers’ pedagogical content knowledge (PCK) has greatly influenced teacher preparation across subject areas. Adapting Grossman’s (1989, 1990) four proposed components of pedagogical content knowledge (PCK), Niess (2005) extended these components to take technology into consideration and further developed the work to consider the process of developing mathematics teachers TPCK (Niess et al., 2009). Several researchers (e.g., Groth, Spickler, Bergner, & Bardzell, 2009; Ozgun-Koca, Meagher, & Edwards, 2010) have used TPCK as a frame for their work to describe the development

of prospective teachers' abilities in using technology in mathematics teaching. Within the context of preparing mathematics teachers, we believe it is most essential to focus on the intersection of technological and pedagogical knowledge within the context of mathematical knowledge. Figure 1 illustrates a representation of technological pedagogical mathematical knowledge (TPMK). Our design approach is based on a model of the relationships among these four knowledge types as depicted in the diagram, where mathematical knowledge is the largest and foundational set.



*Figure 1 Components of Technological Pedagogical Statistical Knowledge (TPSK)*

### **Vision of an Integrated Approach to Developing TPMK**

The *Preparing to Teach Mathematics with Technology* project is focused on preparing teachers to teach several foundational mathematical topics amenable to the use of technology. There are three sets of materials focused on: Data Analysis and Probability (DAP), Geometry (GEO), and Algebra (ALG). Each module includes about six weeks of instructional materials. A central feature in all three books is the inclusion of video clips depicting students' and teachers' work with technology. What follows is an elaboration of our TPMK framework, how we used it to guide the development of materials, and how we embody each of its components in our curriculum.

#### **Mathematical Knowledge (MK)**

We do not intend for our materials to teach all aspects of middle and high school mathematics that teachers should know. However, the materials address topics taught in middle and high school curricula that are amenable to the use of technology tools. Several areas are emphasized to allow teachers to develop understandings that can support their teaching by focusing on:

- Key ideas such as distribution (DAP), congruence (GEO), and functions (ALG);
- Mathematical ideas that cut across all three areas, such as variance and invariance;
- Ways to make curricular connections across DAP, GEO, and ALG to enhance teachers' understanding of mathematics as a connected whole;
- Ways to create mathematical/statistical models of real world phenomena; and
- Developing abilities to create sound mathematical/statistical arguments.

For example, the DAP materials engage teachers in examining distributions graphically and

characterizing the data with such constructs as bins (Rubin & Hammerman, 2006) and a modal clump (Konold & Higgins, 2003) before computing statistical measures. Considering data in the aggregate can allow one to characterize group propensities that can include attention to centers, spread, outliers, clusters, intervals, or residuals. These notions are used in a univariate context to help students consider measures of variation (e.g., residuals, sum of squares), and in a bivariate context when modeling with a least squares line. In accord with the notion of active graphing used by Ainley, Nardi and Pratt (2000), we aim to have teachers consider a statistical model such as a graph or a measure of center as a means to reason with to interpret the models and use in combination with other models to make an argument based on data about the phenomenon under study, rather than merely reporting results of a statistical analysis.

### **Technological Mathematical Knowledge (TMK)**

Although mathematical knowledge is foundational in the TPMK framework, technology tools are used to engage teachers in tasks that simultaneously develop their understanding of mathematical ideas with technology skills so they may experience firsthand how technology can be useful in fostering mathematical/statistical thinking. Through dynamic features such as dragging and linking of multiple representations, technology can help teachers reorganize their statistical and mathematical conceptions. We specifically aim to develop teachers' knowledge in using technology in mathematical ways to:

- Automate accurate computations, geometric constructions, measures, and graphs;
- Visualize abstract concepts through dynamic dragging;
- Explore and coordinate linked numeric, symbolic, graphic, and geometric representations;
- Build mathematical and statistical models, including simulations; and
- Use the internet and networking capabilities to gather data, collaborate on problem solving and share solutions, and identify additional resources needed in solving a task.

For example, in the GEO materials, prospective teachers consider a scenario that involves building a new stadium that will serve three cities. They use dynamic geometry software to create a geometric model of the situation (taking advantage of accurate construction and measurement capabilities), and use dynamic dragging to consider different options (e.g., a location equidistant from the cities, equidistant from the highways, consider populations of the cities). They use the internet to find information about approximate costs, and consider other factors that influence where a new stadium might be built. In the modeling process, they learn about invariant geometric points (incenter, circumcenter) that exist in any Euclidean triangle.

### **Pedagogical Mathematical Knowledge (PMK)**

There are general aspects of pedagogical mathematical knowledge are not dependent on technology, per se, but are more likely to come into play when technology is used in the classroom. We consider these aspects to be:

- Understanding students' potential misconceptions in statistics, geometry, and algebra, and accommodating for differences in learning;
- Supporting students in making statistical/mathematical arguments based on evidence;
- Designing/using/modifying tasks which are exploratory in nature or open-ended;
- Planning for, expecting, and coordinating various approaches and responses to a task;
- Considering the real world aspects of contexts used in tasks; and
- Assessing students' work that involves reliance on new and dynamic ways of representing mathematical ideas.

Throughout the materials, findings from research on students' understandings of statistical and mathematical ideas are used to make points, raise issues, and pose questions for

teachers on how to meet differences in students' learning. Teachers are then often asked to consider pedagogical implications for how they should handle students' responses to a task or how to design a subsequent task that could help students develop a better conception of a mathematical idea. For example, it is not uncommon for middle grades students to believe that since a right triangle is not positioned as they typically see it in textbooks that the triangle is not a right triangle. It is also not uncommon for students to focus on individual data point within a set rather than consider the distribution as a whole. Questions are posed throughout the materials to bring these types of issues to the forefront.

### **Technological Pedagogical Mathematical Knowledge (TPMK)**

The ultimate goal in the preparation of mathematics teachers is to develop the specialized subset of knowledge for teachers representing TPMK. This knowledge encompasses TMK, PMK, and MK. Pedagogical issues and implications come to the fore when considering the particular subset of TPMK. In our materials, we aim to help teachers:

- Understand students' learning and thinking about mathematical ideas with technology;
- Conceive of how technology tools and representations support mathematical thinking;
- Develop a repertoire of instructional strategies for developing lessons with technology; and
- Critically evaluate and use curricula materials for teaching mathematics to make decisions about appropriate and effective use of technology.

We use research results on students' mathematical thinking to guide discussions and tasks. After teachers have engaged in examining a content question with a technology tool (labeled *Engaging with Content*), they are asked pedagogical questions (labeled *Considerations for Teaching*) that are aimed at developing their understanding of how technology and various representations can support students' mathematical and statistical thinking. Questions throughout the materials, and specifically those focused on video clips, can help teachers consider how students might use technology when solving problems and how such an understanding should inform their design of tasks, and choice of technology tools.

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